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Consistent Histories May be Strange, But So is Standard Quantum Theory

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Abstract

Interpretational questions that arise in the Consistent Histories formulation of quantum mechanics are illustrated by the familiar example of a beam passing through multiple slits.

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In the consistent histories formulation of quantum mechanics [1, 2, 3], one classifies certain sets of histories as being "consistent sets", and then identifies values of the decoherence functional as being the "probabilities" of consistent histories. The consistency conditions ensure that, within any single consistent set, the values of probabilities so obtained do have the expected properties. However (as has been known from the beginning [1, 4], and has recently been re-emphasized in [5, 6]), one will not necessarily get sensible results if one combines the probabilities of histories in different sets. Thus it is not clear how to interpret these "probabilities".

Feynman has written [7] that "the *only* mystery" of quantum theory is encountered in the example of a beam passing though multiple slits. Guided by this insight, in this paper we will discuss such an example, to try to illustrate (alas, not to resolve) some of the interpretational questions which are raised in [5, 6].

So we consider, as shown in the figure, a beam of particles incident upon a wall in which there are three slits, which we denote by S_1, S_2, and S_3; each slit may be either open or closed, as we choose. We will not be interested in the entire diffraction pattern, and so we imagine that there is a single detector called D at a fixed location on the far side of the wall. We will only consider those events in which D registers the passage of a particle.

Define A_1 to be the amplitude at D, in the case in which S_1 is open while S_2 and S_3 are both closed. The counting rate in this case is proportional to $|A_1|^2$. Make analogous definitions for A_2 and for A_3, and let $A_{123} \equiv A_1 + A_2 + A_3$. The counting rate when all slits are open is then proportional to $|A_{123}|^2$. In general $|A_{123}|^2 \neq |A_1|^2 + |A_2|^2 + |A_3|^2$, and standard quantum theory does not allow us to say that the particle did go through any particular slit.

If we wish, we can think of S_1 and S_2 as a single object, which we will call S_{12} (it might be helpful, although not necessary, to imagine that there were no separation between S_1 and S_2); below we will refer to this way of thinking as "analysis α". Define $A_{12} \equiv A_1 + A_2$; then the counting rate when S_{12} is open and S_3 is closed is proportional to $|A_{12}|^2$, and $A_{123} = A_{12} + A_3$.

In general, there will be interference between A_{12} and A_3, and standard quantum theory does not allow us to say that the particle did go either through S_{12} or through S_3. Consider, however, the special case in which $A_{12} = 0$. In this case, there is nothing for A_3 to interfere with; $A_{12} = 0$ means that, if S_{12} is open and S_3 is closed, the particle could not reach D. We would surely be tempted, in this special case, to conclude that any particle detected by D must have gone through S_3.

Thus analysis α apparently enables us to say that, if $A_1 + A_2 = 0$,

particles detected by D must have gone through S_3. Of course, instead of analysis α, we could have thought of S_2 and S_3 as a single object (call this analysis β), and then by an analogous argument have said that if $A_2+A_3=0$, particles detected by D must have gone through S_1. Now consider the case $A_1 = A_3 = -A_2$; since $A_1 + A_2 = 0$, we conclude from analysis α that the particle went through S_3, and since $A_2 + A_3 = 0$, we conclude from analysis β that the particle went through S_1. Could both conclusions be correct?

It seems that, when a particle could arrive at D by any of three paths, we can get into trouble by asserting that the particle must have followed one of the paths, even in the case in which the amplitudes of the other two paths cancel. [1] It should be understood that there is nothing that is mysterious going on here *beyond* the mysteries of good old quantum interference. However, those mysteries are so familiar to us that we often forget to be amazed by them. Feynman, in discussing the two-slit experiment [7], stresses that, because of quantum interference, we can not say that particles arriving at D went through either of the two slits. We now see that this remains true even when the amplitude for one of the slits vanishes.

This example, of particles passing through multiple slits, can of course be expressed in the formalism of consistent histories. Let the particle initially be in a pure state; then, with the condition $A_1 = A_3 = -A_2$, the set of four histories in which the particle goes through either S_{12} or S_3, and then is either detected by D or is not, is a consistent set. Consideration of this set corresponds to analysis α discussed above. Within this set, the history consisting of a particle going through S_{12} and then being detected has zero probability (which follows immediately from the condition $A_{12} = 0$, and is what ensures that the set is consistent), and so the conditional probability for a detected particle to have gone through S_3 has value one. Likewise, the set of four histories in which the particle goes through either S_1 or S_{23}, and then is either detected or not, is also a consistent set (corresponding to analysis β). Within *this* set, the conditional probability for a detected particle to have gone through S_1 also has value one. In fact, this example is one of the class of examples considered in [5, 6]; the contradiction referred to in the title of [5] is between the retrodiction that a detected particle

[1] Even if there were only two slits, there could still be similar trouble. Suppose that S_1 were closed (so we need only consider S_2 and S_3), and suppose further that $A_2 = 0$; then we still could not conclude that a detected particle went through S_3. This is because S_2 can be said to have an upper part and a lower part, and (using an obvious notation) it could be that $A_3 = A_{2,upper} = -A_{2,lower}$; then the fact that $A_3 + A_{2,lower} = 0$ could equally-well lead us to conclude that the particle went through $S_{2,upper}$.

went through S_3 and the retrodiction that it went through S_1. In [6] this contradiction is expressed in a different way, which in our example would be the observation that we might conclude (using analysis α) that a detected particle went through S_3, and also (using analysis β) that it did *not* go through S_{23}. Since S_3 is a part of S_{23}, one would have thought that going through S_3 implied going through S_{23}.

If we want to apply the consistent history formalism to a closed system, and *a fortiori* if we want to apply it to the universe, we do not have the option of refusing to accept any conclusion that is not verified by an external measuring device. We are thus faced with the question of under what circumstances and in what sense we should understand the "probabilities" which are assigned to consistent histories. To answer this question, several authors [2, 8, 9] have proposed versions of logic in which statements would only have validity in the context of a given analysis. In our example, this would amount to accepting, for a detected particle, *both* statements "In analysis α, it went through S_3" and "In analysis β, it went through S_1". Then, if someone were to say to us "Now that you have made both analyses, please tell me through which slit the particle *really did* go," we would refer him to [7].

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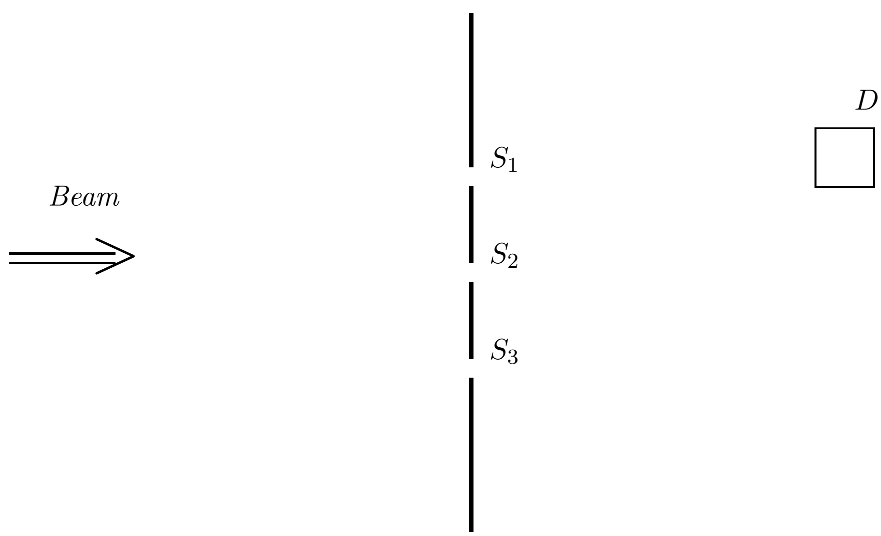

A beam passing through three slits